Analyzing the Mathematical Models behind the Collective Intelligence of Bees

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Abstract

For any system, a mathematical model is a generalization and representation of its features in a very precise manner.

In this paper, we carried out a survey on mathematical models behind the *bees*. The term *collective intelligence*, popularly known as *swarm intelligence* studies the cooperative behavior among the natural species. This self-organized; de-centralized intelligence and cooperation among individuals of a group like ants, bacteria, bees, birds, fishes, termites, wolves, pelicans etc. help them to achieve a common goal like foraging, defending, constructing nests and so on. They frame and follow very simple rules in their entire life cycle, communicate locally in an efficient manner, assist each other in all the situations and many scientific and engineering problems have been solved inspired by these characteristics of swarms.

Keywords: Collective Intelligence, Swarms, Bees

1. Introduction

Swarms are creatures usually live in groups (a colony of ants, bees, flock of birds, school of fishes etc.). A swarm is a population of interacting elements that is able to optimize some global objective through collaborative search of a space (Blesson Varghese and Gerard McKee, 2010). An individual in such a population is always having a tendency of moving towards a direction even in a critical dimension which results in an optimum. These biological systems are dynamic in nature. By following simple and small genetic kind of rules, without any supervision swarms pass much information to their mate about the quality and quantity of food, the distance between the food and the nest and even about the intruders approaching the nest. Many researchers have been started studying these capabilities of swarms and provided solutions to various kinds problems.

2. Characteristics

Swarms are having the remarkable characteristics in nature.

- Flexible: Swarms are enormous in number and so they have the capability of recovering from any tragedy caused by either internal or external entity.
- Decentralized: Swarms are not controlled by any central body.
- Adaptive: Swarms are able to learn new things in case of any changes in their environment and they change their activity accordingly.
- Simple: Swarms follow the small laws within their group without any force.
- Cooperative: Swarms work together to achieve their common goal like building nests, searching for food, defending enemies, transporting food or eggs or dead ones from one place to another in case of any emergency.

By keenly reading such characteristics and environments of bees, the rest of the paper has been organized as follows. In Section 3, we discussed the mathematical concept behind honey bees. In Section 4, we analyzed some problems in engineering which had been suggested with solutions using collective intelligence of bees. In Section 5 we concluded the review of literature.

3. Mathematical Background

If we have a closer look on swarms the laws they follow will lead to a mathematical model which produces effective prototypes to solve our problems. For an instance, the way by which ants find the shortest path between food and nest is used to solve the network routing problems, the manner by which the flocks of birds fly on the sky is employed in local and global searching algorithms, the approach by which tasks are allocated in the insect colonies is helpful in effective task allocation in computing systems and the style by which the bees stores nectar, eggs, larvae in the hive is an inspiration for effective clustering of data. These characteristics help the researchers in formulating new methodologies and devices to propose and maintain communication systems and networks which naturally adaptive to dynamic environments with other competencies like scalability, diversity and identity. In this section, we analyzed such mathematical concepts behind bees.

3.1 Bees

Honey bees are collective insects and they always live in group under a composite and cooperative society (Diomar Cristina Mistro, LuizAlberto Díaz Rodrigues and Wilson Castro Ferreira Jr., 2005). In the work (Pham D T, Ghanbarzadeh A, Koç E, Otri S, Rahim S and Zaidi M., 2006) the authors introduced how the cooperative and intelligent behavior of bees could be used for solving the engineering problems. There are three kinds of honey bees, viz. queens, males and worker bees. A queen produces eggs; males mate with queens and the worker bees guard their hive and collect honey. Bees in a colony may vary in numbers with thousands of creatures. They communicate among each other by spreading the chemical pheromone and dancing. In each hive, there is a specific part which is allocated only for communicating the information via dancing. From a particular hive, the foraging starts with sending worker bees to search randomly for area in which there is a scope for nectar. Once return to the hive, the worker bees lay down the nectar in order to prove that the worth of the nectar is above a threshold, they start dancing. It is a way of communicating the information about excellence of nectar they found, the distance between the nectar source and their hive and even the direction in which they found the food source to their mates. The entire group now evaluates the quality of food by comparing with each other and distance they need to travel. After finishing dancing, the worker bees start for gathering nectar with other worker bees that wait in the hive. Once gathered the nectar or pollen from a flower patch, the group evaluates the intensity so that a good food source can be indicated to other groups while depositing the nectar.

The probability value associated with food source p_i is calculated by the equation:

$$p_i = \frac{fit_i}{\sum_{n=1}^{N} fit_i} \qquad (1)$$

Where, fit_i is the fitness value of bee_i and *N* is the total number bees employed for foraging In this approach, bees communicate the information, cooperate with others.

D.T. Pham et.al had derived an algorithm as given below, which is the base for Bee Colony Optimization (BCO) techniques.

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Step 1: Initialize population with random solutions.

Step 2: Evaluate the fitness of the population.

 Step 3: While (stopping criterion not met) repeat the following:

     //Forming new population.

      Step 3.1: Select sites for neighborhood search.
     Step 3.2: Recruit bees for selected sites (more bees for
     best e sites) and evaluate fitness values.

     Step 3.3: Select the fittest bee from each patch. Step
     3.4: Assign remaining bees to search randomly and
     evaluate their fitness values.

Step 4: End While.
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BCO algorithms can be used to solve both combinatorial and functional optimization problems. They had also proved that this BA converged to the best results (both minimum and maximum values) without fell in local minima.

Africanized honey bees (*Apis mellifera adansonii*), popularly known as killer bees, are a crossbreed of European honeybees and African honey bees. These bees are too violent, which are capable of attacking the Europeans' hive, by killing the queen they can have their own African queen in that hive (S S Schneider, T Deeby, D C Gilley and G DeGrandi-Hoffman, 2004). Due to some natural disasters like fire, heavy rain etc., the queen bee leaves the hive and moves to a new place to make a new hive. The entire colony movement from one place to the other with better resources is called as *taxis (*Edelstein Keshet L, 1988). This dispersal, also called as *absconding* is the most frequent swarming behavior of these African Honey Bees. Reproductive swarms constantly move away from their parent's in order to avoid competition between them. Mostly, they start moving from their own hive when the number of bees increases and there is insufficient space and food. This behavior is known as *neighborhood repulsion*, which exists among AHBs. (Seeley T.D., 1995). In their work (Diomar Cristina Mistro, LuizAlberto Díaz Rodrigues and Wilson Castro Ferreira Jr. 2005), the authors proposed a mathematical model for these swarming behavior of AHBs. The population dynamics of the colonies has been defined in terms of a *discrete time model* as below:

$$N_{t+1} = \sigma N_t + N_t \exp[\, r \left(1 - \frac{N_t}{K}\right) \qquad (2)$$

where,

N_t : density of the colony in time 't'

□ : survival fraction from generation 't' to 't+1' K

: threshold / saturation value of population density

r : reproductive rate

The spatial rearrangement of colonies found in the space interval [a, b] at time 't' was described by the integral operator method [10] as below:

$$\int_a^b N_t(x)dx \qquad (3)$$

The density of the population changes due

- to advent of colonies from the old generation / absconding ones
- to advent of colonies from reproductive swarms
- to colonies that stay in their novel sites

By considering the absconding generation from one site to the other, the new colonies generated the following mathematical model is derived finally.

$$N_{t+1}(x) = (1 - p_a)\sigma N_t(x) + p_a\sigma \int_{-\infty}^{+\infty} k_a(x,y)N_t(y)dy + \int_{-\infty}^{+\infty} k_r(x,y)f\big(N_t(y)\big)dy \quad (4)$$

where, the first term in the above equation represents the density of the colony stay at site 'x' after some p_a abscond from 'x', the second term represents the densities of the colonies which absconded from 'y' and landed in 'x' and the third term represents the new colonies generated at 'y' and moved away from

'y'.

Simply, the dynamic population of AHBs can be given in terms of a *non-linear function* as:

$$N_{t+1}(x) = \varphi[N_t](x) \quad (5)$$

By considering reproduction parameter, carrying capacity, survival rate, absconding probability, Weibull Dispersal parameter and Gaussian Dispersal parameter, the simulation results proved the characteristic *non-linearity* in AHBs. The diffusion of the honey bee colonies is giving an idea about solving non-linear problems in one angle, in another way the structure they used for building their hives is providing an inspiration for pattern formation and clustering. A proposed model (Camazine S., Sneyd J., Jenkins M. J., Murray J. D., 1990) produced the patterns for the storage of nectar, pollen and the offspring in the honey bee hives. Bee hives are closed structures, which are the array of hexogen cells. Bees use these cells to store up honey and pollen, to accumulate different stages offspring like eggs, larvae and pupae (Michael J. Jenkins, James Sneyd, Scott Camazine and J. D. Murray, 1992). The worker bees bring the pollen and nectar from the flowers and fill the cells in the hive. They deposit the substances in the cells which are unfilled or partly filled with the same stuff. Almost in all such colonies similar pattern appear like at the bottom deposition of brood and at the bottom deposition of honey which are divided by a collection of pollen.

N H Fefferman and P T Starks presented a mathematical model of a definitive hypothesis for swarming in honey bees. It has been discussed that 'swarming' occurs when the parent colonies approach the

'replacement stability'. The following are identified as the basis for honey bees swarming:

- increase in *colony size* : the count of worker bees reaches a particular threshold value
- decrease in *brood comb* : the count of eggs laid by the queen bee increases over time which leads to an obstruction in the nest for the broods
- increase in *young bees* : the proportion of younger workers to the older ones increases
- decrease in *communication* : due to the above three reasons the messenger bees are unable to contact their queen bee frequently

For each of the above hypothesis, the trigger A_i has been defined. For example if the colony size increases over the period of time and reaches a threshold value T, then the movement of the parent colony is triggered, which was given as below:

$$A_i = \begin{cases} 0 \; if \; \{ (C_i < T) or (A_{i-1} > 9)\} \\ 1 \; if \; \{ (C_i \geq T) or \; (H_i = 1 \; \& \; i = I_{s1} + 1)\} \end{cases} \quad (6)$$

Here, C_i : the size of the colony in the day i,

T : the threshold value for the size of the colony,

A_i : the decision variable which triggers the decision to swarm,
H_i : the number of swarms that left the colony in day i,

I_{si} : the day of the growing season when the i^{th} swarm leaves the parent colony.

Thus it is necessary to identify the reasons for queen rearing in order to maintain the honey bee colony effectively.

4. Scope

Many chemical and physical systems are characterized by formation of patterns, clusters and aggregates, or phenomena such as wave and pulse propagation (Leah Edelstein-Keshet., 2001). Similarly when we look into the biological systems, such as bees foraging and swarming are also in the form of the above said phenomena. It has been discussed about various number of routing protocols for wireless sensor networks that have been built up based on the swarm intelligence concept, specifically by considering the idea of the ant colonies and bee colonies (Muhammad Saleem ,Gianni A. Di Caro, Muddassar Farooq, 2011). A novel taxonomy for routing protocols in wireless sensor networks has been introduced and it has been used to classify the various protocols surveyed. Researches (Dervis Karaboga and Bahriye Basturk, 2007) proposed that ABC was giving better results for numeric optimization problems. They have also proved that ABC algorithm was yielding enhanced results or equal results when compared with other evolutionary algorithms by taking benchmark functions for experiments (Dervis Karaboga, Bahriye Akay, 2009). Apart from regular parameters (population size, number of evaluations), only one control parameter, *limit* had been used by them. Thus it was proved that if the control parameters are lesser in numeric computations, then ABC can be deployed to obtain the optimized results very quickly. In their research (Fei Kang, Junjie Li and Haojin Li., 2013), they suggested a hybridized methodology by combining ABC algorithm with Hooke-Jeeves pattern search algorithm to solve numerical optimization problems with accurate results and faster convergence towards the solution. To conclude *Artificial Bee Colony* algorithm is not only suitable for solving single optimization problems but also for multiobjective optimization problems (S.N. Omkar, J. Senthilnath, Rahul Khandelwal, G. Narayana Naik and S. Gopalakrishnan, 2011). The authors proved this by adapting ABC in solving composite design problem. They formulated the composite design problem into multiple objectives (weight and cost of laminated structure) problem and shown that ABC was giving promising results by minimizing weight and cost of the design.

5 Conclusion

In this paper, we have analyzed the social behavior of honey bees and the mathematical equivalences for their activities. The concept of *artificial bee colony (ABC)* is an approach used now-a-days by researchers to solve various optimization problems. Non-linear, unimodal, multimodal optimization problems can be simply resolved by using ABC algorithm. Based on the objectives of the problem, the control parameters might be selected minimum in number when compared to other collective intelligence algorithms. It can be proved in the near future that any kinds of optimization problems can be solved using ABC.

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